Table 2 Computation error of eigenvalues

Order of eigenvalue	Error, %
First	0.00
Second	0.00
Third	0.00
Fourth	0.00
Fifth	5.01

Table 3 Computation error of eigenvector derivatives

Order of mode	Error, %	
	Synchro	Shift
First	0.49	0.00
Second	0.56	0.00
Third	1.8	4.03
Fourth	2.11	17.58

Table 4 Computational times for eigenpairs and their derivatives

Method	Computational time, s
Synchro Lanczos	98
Shift Lanczos	148
Ratio	66.15/100

The errors of the lowest four eigenvector derivatives are listed in Table 3 (compare with the results calculated by Nelson's method).

When the frequency-shift Lanczos method is used to compute the eigenvector derivatives, select the initial vector so that the order of reduced equation is one, i.e., $T = \alpha_1$. The accuracy of the eigenvector derivatives can be kept at less than 1%. The errors of the third and fourth eigenvector derivatives listed in Table 3 are larger than 1% because of the effect of the error of eigenvector. When a high-precision eigenvector is used to compute eigenvector derivatives, lower accuracy is obtained by the synchro Lanczos method than by the frequency-shift Lanczos method, but the error is still less than 1%. The frequency-shift Lanczos method is more sensitive to the error of eigenvector than the synchro Lanczos method. Computational times are listed in Table 4.

About a one-third savings in computational time over the frequency-shift Lanczos method is observed when using the synchro Lanczos method. The frequency-shift Lanczos method can effectively reduce the order of the system, but it needs to solve a set of system-size equations to select the initial vector for each mode of interest. The synchro Lanczos method can arbitrarily select the initial vector. The result is obtained by selecting an initial vector $\bar{v}_1$ consisting of random data and $\bar{v}_1' = 0$. When selecting $\bar{v}_1$ consisting of the diagonal elements of K and $\bar{v}_1'$ consisting of the diagonal elements of K', there is little difference in the results obtained by the two methods.

Conclusions

The improved Lanczos method can be used to compute the eigenpairs and their derivatives simultaneously. Synchro calculation is the major peculiarity and advantage of this method. It can use not only the final results but also the median results obtained in the preceding eigenanalysis. Higher efficiency is observed when using the present method than when using the frequency-shift Lanczos method. However, as the number of design variables increases, the derivative computation of median matrices increases with each recursion, and the superior efficiency of the present method decreases or even reverses itself. However, synchro calculation provides a new idea for sensitivity analysis.

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Virtual Member Method for the Analysis of Frame Structures with Damping Joints

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Introduction

A THREE-DIMENSIONAL frame structure is assembled from one-dimensional members. Vibration of such a structure can be referred to as elastic disturbances propagating in the structure. The elastic waves propagate along members but scatter and reflect at junctions. From this point of view, a traveling-wave model of the structure can be constructed. Because the model is directly obtained from exact boundary conditions at junctions and partial differential equations that govern waveguide motions, the traveling-wave model can provide more accurate dynamic properties of the structure. Furthermore, it is also very convenient for the consideration of the vibration control and the building of a precise local model for a structure.[1–8]

Von Flotow[1] and von Flotow and Schafer[2] presented the traveling-wave model of large flexible spacecraft structures and studied the vibration control problem based on the idea of wave absorbing and wave isolation. Miller and von Flotow[3] and Beale and Accorsi[4] analyzed the power flow in structure networks and gave the energy transmission paths. Vibration control based on a traveling-wave model was also studied by MacMartin et al.[5] and Matsuda and Fujii.[6] Other works also deal with local models, such as complicated junctions[7] and complicated members.[8]

Modern large spacecraft structures (LSS) have been becoming more and more complex and flexible. Dynamic behaviors of LSS need to be accurately controlled to satisfy mission requirements. Because it lacks sufficient damping, such a structure will cause the lessened effectiveness of many envisioned active control methods.[9] For this reason, the damping treatment of a structure is very important. Some of the most popular damping treatment methods are damping layers, damping joints, and damping dashpots.[10]

Damping joints have the advantages of being simple and easy to realize. They can also obtain a good vibration suppression effect.[11] Damping enhancement using active and passive joints has been studied by some researchers.[12,13] In these works, models of the joints are developed mainly in modal space, and only very simple structures, such as beams, have been considered. In practical engineering, the configuration of a joint may be very complicated, and so it is desirable to develop a technique to deal with an arbitrary complex frame structure with complicated joints.

Received Jan. 15, 1997; revision received Nov. 15, 1997; accepted for publication Jan. 2, 1998.

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This Note discusses the dynamic properties of linear damping joints in a traveling-wave formulation. A traveling-wave model for frame structures that is similar to that in Refs. 3 and 4 is first presented. A linear damping joint connecting two members in an identical line is studied as a basic form of a damping joint. A joint with other configurations is defined as a complicated joint in the present Note. The concept of a virtual member is proposed to convert the complicated joint to a rigid junction and damping joint that connects two members in an identical line; thereby the analysis for structures with complicated damping joints can be greatly simplified.

Traveling-Wave Model for Frame Structures

A frame structure is assembled from members through junctions. Axial, torsional, and flexural waves propagate along the members, which are one-dimensional waveguides. Each member must have continuous mechanical impedance, and all of the forces and the displacement boundary conditions are imposed on the junctions.

The local coordinate system for a member is shown in Fig. 1a. Nodes 1 and 2 are called the left and the right nodes, respectively. The Fourier transformations of different kinds of waves are referred to as wave modes.[3,4] The wave mode vector of an arbitrary cross section is defined as $w_m = [w_l^T, w_r^T]^T$, where w_l and w_r are the left-forward and right-forward wave modes, respectively. The dynamic model of a member is described by Eqs. (1–3):

$$\begin{Bmatrix} u_m \\ f_m \end{Bmatrix} = Y(x, \omega) w_m \tag{1}$$

$$w_m|_{(x_2,\omega)} = \xi_m(x_2, x_1, \omega) w_m|_{(x_1,\omega)} \tag{2}$$

$$\begin{Bmatrix} U_m \\ F_m \end{Bmatrix} = \lambda_m \begin{Bmatrix} u_m \\ f_m \end{Bmatrix} \tag{3}$$

Equation (1) is the state transform equation, where $Y(x, \omega)$ is a state transform matrix, and u_m and f_m are the displacement vector and the inner force vector at a given member cross section, respectively. Equation (2) is the member transmission equation, in which the member transmission matrix $\xi_m(x_2, x_1, \omega)$ describes the variation of magnitude and phase when the traveling waves propagate along the member. Equation (3) transforms the displacements and inner forces from a local coordinate system to a global coordinate system for taking the boundary conditions into consideration.

External forces and the displacement boundary conditions are imposed on junctions, and the conditions of force equilibrium and displacement continuity should be satisfied also at the junction. A schematic diagram of the junction is shown in Fig. 1b. Suppose that N members are connected to the junction and that on each member there are incoming wave modes w_{ni} and outgoing wave modes w_{no}. For a rigid junction that connects the members rigidly, from the conditions of force equilibrium and displacement continuity, it is obtained that

$$\begin{gathered} \alpha \sum_{i=1}^{N} F_i + \beta[U_1 - \delta] = Q \\ U_1 = U_2 = \cdots = U_N \end{gathered} \tag{4}$$

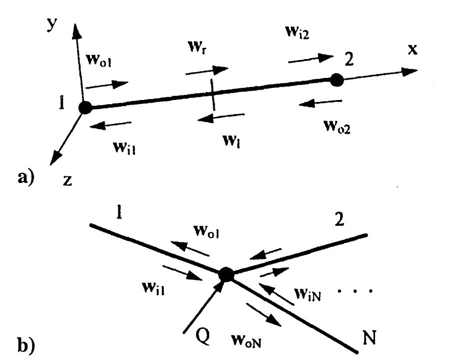


Fig. 1 Schematic of a) a member and b) a junction.

where δ is prescribed junction displacements, Q is prescribed junction loads, and α and β are boundary condition selection matrices. Let $W_{no} = [w_{1o}^T, w_{2o}^T, \ldots, w_{No}^T]^T$ and $W_{ni} = [w_{1i}^T, w_{2i}^T, \ldots, w_{Ni}^T]^T$; from Eq. (4), the junction scattering model can be obtained as

$$W_{no} = S_n W_{ni} + G_n R_n \tag{5}$$

where $R_n = \beta\delta + Q$, S_n is a junction scattering matrix, and G_n is a junction generation matrix.

Let W_i and W_o denote the system incoming and outgoing wave mode vectors, respectively; the system scattering equation can be assembled from Eq. (5) with the superposition method similar to that of the finite element method:

$$W_o = SW_i + GR \tag{6}$$

where S is a system scattering matrix, G is a system generation matrix, and R is a vector of prescribed loads and displacements.

Assemble Eq. (2) for all members; it gives

$$W_o = TW_i \tag{7}$$

where T is a system transmission matrix. The terms W_i and W_o can be solved for specified R from Eqs. (6) and (7), and then inner forces and displacements at an arbitrary cross section are obtained from Eqs. (1) and (2). Because the preceding model is developed in the frequency domain, the result is the frequency response when external loads have a unity amplitude.

Wave Model for Damping Joint

A damping joint is a special kind of junction at which there exists energy dissipation. The energy dissipation is realized through the relative motion between members at the damping joint. The continuity condition is no longer preserved for some displacements, and this can be replaced by a back-restore force equation at the joints. The joint may have various configurations. The most simple one is a joint that connects two members in an identical line; such a joint can be called a simple joint. For the joint with other configurations, which is defined as a complicated joint, the scattering model of the joint depends greatly on its configuration. In the following paragraphs, the simple joint is first analyzed. For a complicated joint, virtual members are introduced to simplify the configuration of the joint; thus the joint model can be easily constructed. An Euler–Bernoulli beam is considered here to show the joint dynamics.

A simple damping joint that connects two members in an identical line is shown in Fig. 2. The external force vector q is also divided into two parts, i.e., $q = q_1 + q_2$, where q_1 and q_2 are external forces applied at the end of elements 1 and 2, respectively. For the sake of convenience, only one relative displacement is considered here.

Let $r = r_2 - r_1$ denote relative displacement; let r_1 and r_2 be displacements of members 1 and 2 at the joint, respectively; and let f_r denote the restore-back force that acts on member 1; then the dynamic property of a linear damping joint can be written as

$$f_r = c_d \dot{r} + c_k r \tag{8}$$

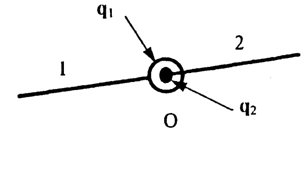
where c_d and c_k are damping and stiffness coefficients, respectively. For a beam member, the displacement vector is $u = [u_x, w_y, w_z, \phi, \psi_y, \psi_z]^T$, u_x and w are transitional displacements, and ϕ and ψ are angles of rotation. Subscripts x and y denote the point direction for w and also denote the rotating axis for ψ. Different kinds of joint models can be obtained when r_1 and r_2 represent one of $\{u_x, w_y, w_z, \phi, \psi_y, \psi_z\}$, and f_r is the corresponding inner force.

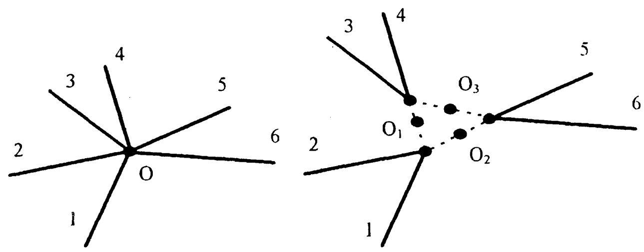
For a junction with two rigidly connected members that are in an identical line, Eq. (5) is also held in the local coordinate system

$$f_1 + f_2 = q \tag{9a}$$

$$u_1 = u_2 \tag{9b}$$

Fig. 2 Simple damping joint that connects two members in an identical line.





a) Configuration of the joint b) Simplified configuration

Fig. 3 Damping joint connecting more than two members.

For the case of a damping joint, Eq. (9b) is no longer correct. Equation (8) should be substituted into Eq. (9b) to replace the equation about r_1 and r_2. For example, if r_1 and r_2 denote the rotation angle of members 1 and 2 at the joint, i.e., $r_1 = \psi_{z1}$, $r_2 = \psi_{z2}$, and $f_r = M_{z1}$, the sixth equation of Eq. (9b) should be replaced by

$$M_{z1} = c_d(\dot{\psi}_{z2} - \dot{\psi}_{z1}) + c_k(\psi_{z2} - \psi_{z1}) + q_{16} \tag{10}$$

The resulting equations are then written in the wave mode form to get the joint scattering model, which can be used in assembling the system equation.

To analyze a frame structure with complicated joints, the concept of the virtual member is introduced here to convert a complicated joint into a set of simple joints and rigid junctions. The virtual member has the same parameters as those of the member it connected, except that it has a zero length. The joint property is separated from the complicated joint to simple joints by virtual members, so that the virtual member has no contribution to the system dynamics but can simplify the configuration of the complicated joint to the available junction models. Wave modes also propagate along the virtual members.

A complicated joint is simplified with the following procedure. Members attached to the junction are first divided into groups, each group is formed by some rigidly connected members, and these groups have relative displacements to each other. A simple damping joint that connects two virtual members in an identical line is used to connect each pair of the groups. The restore-back forces between each pair of the groups are represented by the damping joint connecting the virtual members. This procedure can be described more clearly by the following example.

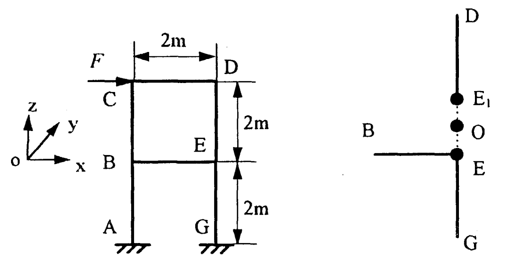
The complicated joint shown in Fig. 3a is presented as an example. There are six members attached to the junction O. Members 1 and 2 are rigidly connected, members 3 and 4 are rigidly connected, and members 5 and 6 are also rigidly connected. Relative displacements exist between each pair of these three member groups; then three simple joints are needed to simplify the complicated joint. The simplified configuration is shown in Fig. 3b. The dashed lines represented virtual members. Junctions O_1, O_2, and O_3 are simple joints, whereas others are rigid junctions.

Numerical Example

A frame structure is shown in Fig. 4a. All of the members are rods with radius of 0.01 m and mass density $\rho = 7800$ kg/m^3. The material damping property is taken into account by complex modules $E_C = E_R(1 + i\eta)$, where $E_R = 2.0 \times 10^{11}$ N/m^2 and the loss factor $\eta = 3.0 \times 10^{-4}$. External load F is applied at junction C in the x direction. Member DE is connected to junction E with the damping joint. Only the rotation angle about the y axis is no longer continuous, and the corresponding restoring moment is $M_y = -c\dot{\psi} - k\psi$, where ψ is a relative rotation angle.

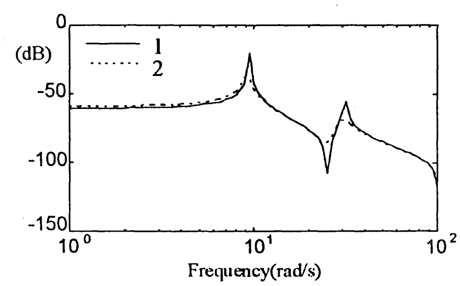
A simple joint connecting two virtual members is added between member DE and junction E (Fig. 4b). The joint dynamics are imposed on joint O, and then E and E_1 are rigid junctions. Member OE and OE_1 are virtual members that have a length of zero.

The structure in which all members are rigidly connected is analyzed first. Structure transfer properties (the solid line in Fig. 5) have a high resonance peak at the natural frequencies of the structure. Figure 5 also gives the transfer properties when the damping joint is employed (dotted line). The corresponding joint parameters are $c = 100$ N · m · s and $k = 0$ N · m. The results show that, when the damping joint is introduced to dissipate the vibration energy of the structure, the vibration amplitudes at natural frequencies can be reduced significantly.

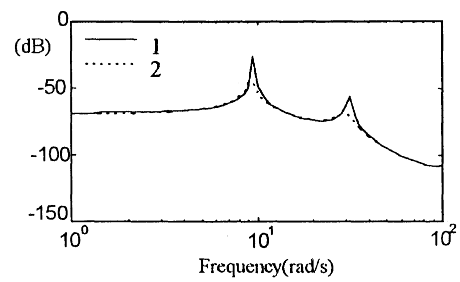


a) Schematic diagram of structure b) Virtual members OE and OE_1

Fig. 4 Frame structure in example.



a) $|w_{xC}/F|$



b) $|w_{xB}/F|$

Fig. 5 Structure transfer properties: 1, rigidly connected, and 2, damping joint.

Conclusions

A joint is a kind of connection commonly used in engineering and has various configurations. The analysis of a structure that has complicated joints is always a difficult problem. The concept of a virtual member is proposed in this Note to convert the complicated joint to simple joints and rigid junctions; thus the structure can be analyzed with the existing traveling-wave model. The result has shown that this is a very simple and effective method. The idea of a virtual member provides us with an approach to which, for a complex system, some virtual elements can be added. Although the virtual element does not change the dynamics of the system, it can more clearly represent the configuration of the system. As a result, the system can be analyzed with the existing method in a routine way.

Acknowledgment

The research was sponsored by the National Natural Science Foundation of China.

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